Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

# THE PRESENT STATE OF THE QUESTION REGARDING THE FIRST PRINCIPLES OF THEORETICAL SCIENCE.

By JOSIAH ROYCE.

(*Read April 19, 1906.*)

I venture to use this opportunity to call attention to the existence and to the spirit of certain researches in which a good many well-equipped students of science are now taking part, and in which I myself, although very ill prepared for the work, have already tried in a very modest way to take some part myself. These researches interest primarily logicians, and to some extent mathematicians. They have a relation, however, not only to general philosophy, but also to the interests of a good many students of the special sciences. Let me try briefly to indicate the problems which give rise to such researches.

## I.

Science, as we all know, has two aspects, namely, that aspect which is concerned with discovering and reporting facts, and that aspect which is concerned in constructing and applying theories. A scientific theory is a body of assertions connected together by processes of logical reasoning, and so chosen as to be of use in displaying the rational connections of facts, and in predicting facts which have not yet been observed. The extent to which theories are of use for the work of a given science varies very greatly with the stage of evolution which the science has reached, with the character of the subject matter and with the interests which control our study of the facts in question. Celestial mechanics furnishes an instance of a very highly developed theoretical branch of science. The extent to which theory is significant and successful in any one science, as for instance in biology or in chemistry, is in case of each such branch of scientific inquiry a kind of test of the stage which the science in question has reached in its evolution. In the history of

a science the premature prominence of theoretical constructions leads to a neglect of facts or to a too easy contentment with an insufficient collection of facts. When a science, however, is already highly developed but is also rapidly growing, the search for new facts is commonly guided by more or less highly developed theoretical interests, and is directel by presuppositions, by hypotheses, by questions, which have come to mind in consequence of reasonings due to theories.

In the normal case of a science in which theories play an important part, a scientific theory takes the form, first, of the statement of a set of principles, or of relatively fundamental propositions, which the theory treats, at least, provisionally, as true. Secondly, the theory consists of the logical development of a set of consequences, which follow from these principles and so will be true in case the latter are true. These consequences may be reached, and in the case of the most highly developed theories, are reached, by mathematical computations. In its application to the work of the science, the theory becomes useful, in so far as its results can be compared with the particular facts of experience, or can be tested by seeing how far they lead to successful predictions.

A theory whose results disagree with facts has to be amended accordingly, but in many cases may be adjusted to the facts by alterations which leave its main principles intact, and which involve only minor modifications. When a theory succeeds up to a certain point, but leaves some facts indeterminate, it frequently gives rise to hypotheses concerning phenomena as yet unobserved, and in this sense may prove a guide to investigation. There are well known and important cases where a theoretical computation disagrees for a time with actually observed facts, but where the discrepancy can be shown to be due to the non-recognition of certain facts which, so soon as you take them into account, enable the original theory to apply with reasonable accuracy to the whole system of facts in question. In some cases improved methods of calculation, or other purely logical developments of theory itself, suffice to remove discrepancies; and such cases furnish very persuasive tests of the value of the theory in question.

If you look towards the world of facts, as experience shows them

to you, the principal use of a theory seems to lie in two things. First, a theory, if successful, enables you to give an economical description of a vast number of facts. Secondly, a theory usefully guides your search after new facts, and in particular your predictions, and the practical activities by means of which you apply your science to the study of new cases. The common mind often opposes theory and practice. But every enlightened student is aware how large a part theory plays, in those cases where theory is possible, as a means towards guiding the practical applications of a science to the various arts. Thus without astronomical theory the application of astronomy to navigation would remain very limited; because one who has to apply observations of the heavenly bodies to the work of the navigator must accomplish his application by means of definite processes of computation. Such computations can be reduced to precise rules only by means of considerations which belong to the theoretical side of the science. So long as, for the Babylonian astrologers, astronomy remained a mysterious branch of empirical natural history, computations could have only a limited scope. It is astronomical theory, not to be sure the whole of astronomical theory, but a certain limited portion of it, which gives to the navigator's computations a uniform and controlable character. Economical description, controlable application to the search for new facts, and to the practical uses of a science, these are the characters which a scientific theory must possess in order to meet the requirements which the real world makes upon it.

But these requirements cannot be met unless the theory possesses a certain coherent logical structure. This structure might in general be possessed, almost or quite equally, by a great number of different theories whereof only one happened to be true to the facts. Nevertheless, although the internal logical finish of the structure of a theory is by itself no guarantee that the theory is useful in describing or predicting facts, such logical structure is a condition sine qua non of a good theory. A natural question arises as to what constitutes this internal logically coherent structure to which a highly developed theory must conform. The question so stated may appear at first sight very vague and indeterminate. A theory, you will say, must make use of principles which it provisionally assumes to be

true. It must then develop the consequences of these principles. It must be logically accurate, and as full in its development of consequences as the application which is to be made of the theory seems to require. This, it would seem, can in general be said. But with this vague generality the theory of what constitutes a good theory would appear at first sight to be completed.

Yet a moment's thought will show, that we all pretend to know more about the structure towards which highly developed theories tend, than this first generalization would make manifest. For instance, it is a comment which has become commonplace, that, wherever quantitative conceptions are possible, theories whose first principles can be expressed in quantitative form, have a formal advantage over theories which have to be expressed in non-quantitative terms. Some portions of our empirical world are subject to measurement. Measurement in practice gives results which vary within the limits of error, and which are therefore inexact. A theory which is to be just to any highly advanced state of knowledge regarding measurable facts, must make use of principles which involve provisionally assumed relations of quantities. One advantage which a quantitative theory can then possess lies in the very fact that its provisionally assumed principles may be stated with an exactness which empirical measurements never reach. In other words, the very incapacity of our theory to account for the variations, and for the inexactness of any single process of measurement, may be an advantage in the development and in the further application of the theory. Assuming exact relations, and invariant relations, where the actual measurements of observers show a considerable range of uncontrollable variation, the theory may enable computations to be made, in terms of which the work of measurement may be guided, and the essential and unessential elements of experience may be distinguished. Cases of this sort suggest that the structure of theories is subject to certain logically definable laws which are somewhat independent of the precise degree to which in a given case the theory in question can be verified. In other words, while the true theory, in the sense of the theory that agrees with the observed facts, is indeed the ideal, one may be able to judge the value of a theory in advance of knowing whether it is true of not, in so far, for instance, as a quantitative

theory is preferable to a non-quantitative one, and in so far as exact theoretical interpretations are preferable to inexact ones.

This very commonplace instance suggests where lies the logical problem regarding the internal structure of theories. What does one mean, for instance, by a quantitative theory? In order to answer this question one must know what one means by quantity. Why are quantitative ideas more useful than non-quantitative ideas? Wherein lies the logical difference between conceptions of quantity and other conceptions? Is the notion that quantitative conceptions stand alone amongst possible scientific conceptions, in their peculiar possession of exactness, and of a capacity to be submitted to precise and entensive processes of deduction, is this presupposition itself well founded? Are there other concepts which are logically as exact as the quantitative concepts, which are as capable of being subjected to elaborate processes of a deductive character? If so, are there other regions than those of the sciences of measurement in which highly developed theoretical finish is possible? May the science of the future come to use other than quantitative theories in dealing with regions of nature or of mind where measurement proves to be unattainable, or inexact? How will the non-quantitative theories, in so far as they can be developed, stand related to the quantitative theories? What is it that makes certain concepts adapted to furnish a wide range of unexpected results, which can be reached deductively, and by exact devices of thinking, although these results cannot readily be seen at a glance, by merely inspecting the conceptions in question? How can mere deduction lead to an infinite number of unexpected results, as is often the case in the exact sciences? Do the possible conceptions which the human mind can frame, and can lay at the basis of theoretical constructions, form anything like a closed system? In other words, is the range over which our theoretical constructions vary simply limitless, and indeterminate, or is it, even if infinite, still in some way itself determinate, so that one can name certain fundamental concepts which every theory must use, or from which every theoretical construction must make a selection, even in defining its provisionally assumed principles? In other words, are there first principles of scientific theory? Are the ideas which we can use in defining our provisional hypotheses, in

initiating processes of logical deduction, ideas of which some general and thoroughgoing account is possible, so that, although we cannot predict the facts of the natural world, we can predict the forms in terms of which we shall always be obliged to think the rational connections of these facts in case we form any theory at all? Are, then, the internal conditions of theoretical science, the logical possibilities upon which such a science depends, of a determinate range, and of a knowable character? Such are the problems which are suggested when we begin to inquire as to the logical position which quantitative theories hold amongst the various types of theories which are logically possible.

The questions thus suggested are obviously of the most fundamental importance for any one who is interested in understanding the workings of science. Science depends upon finding facts; it certainly also aims at the controlling of facts. The control which is here in question may either mean the technical mastery of facts, the power to produce them at will, or it may mean the prediction of facts. But either kind of control is possible only in so far as we possess something of the nature of a theory. And a theory involves the construction and control, and logical linking of concepts which have to be of our own making. Therefore, the study of the types of concepts which we can construct and control and link, the study of the forms and linkages which the nature of our thought makes possible, is surely as serious a study, as the direct study of the facts which we can hope to control through the use of our intelligence. The pursuit of useful knowledge surely includes in the end a knowledge of those logical processes of thought whereby we come to make an intelligent use of facts.

## II.

The result of such considerations is that a science is needed which I may provisionally call the morphology of theories. This science is a branch of logic. And it is to this science that I now call your attention.

So far it is easy to define our problem, and to see that if solvable, it must be an important problem. What the student unacquainted with modern logic will find doubtful may be the assertion that such a problem can at present be fruitfully studied. If you define this

study of the first principles of theoretical science as a branch of what is called logic, it was until recently the fashion to say that since Aristotle logic has made no progress; that that marvellous thinker had already seen nearly all of what the human mind can see regarding the structure of our thinking processes, and regarding the way in which we can use principles provisionally assumed, for the purpose of drawing conclusions from them. The principal addition that was supposed to have been made to logic since Aristotle was confined, according to this view, to a study of that inductive logic which is concerned rather with the application of our thinking processes to the discovery, the collection and the arrangement of facts, than with the structure of our thinking process itself. I wish to call attention on this occasion to the fact that this familiar assertion concerning logic and concerning its stagnation since Aristotle, in no longer true. We are today in the midst of a very vigorous and many-sided movement which interests the students of several different sciences, and which involves a rapid advance towards an answer to those very questions which I have just enumerated. We are today in a way to grow very rapidly in our comprehension of the range, of the varieties, and of the logical nature, of the fundamental conceptions upon which all theoretical science depends. We are no longer confined to the commonplace observations just cited regarding the peculiarly advantageous character of quantitative concepts and theories. We begin to know *why* the concept of quantity has the logical usefulness that it possesses. And as we come to know this, we see that the concept of quantity is one only amongst the exact and definable fundamental concepts upon which scientific theory depends. We discover that even the quantities get their logical usefulness for purposes of scientific theory from certain characters which they share with a very large number of other concepts, namely from their character of being capable of serial arrangements. and from their further character of constituting what is now called a group, with reference to certain specific operations. The series concept and the group concept thus obtain a logical place amongst fundamental concepts which permits us at once to view the quantitative theories as a special instance only amongst an infinite, but again perfectly determinate range of possible theories, some of which have

already their place in certain of the sciences, while other exact, and equally fruitful, although non-quantitative theories, are likely to become of definite use in the science of the future. We are, therefore, already on the way vastly to enlarge, but on the other hand much more precisely to define our concept of what constitutes an exact scientific theory. We are on the way towards understanding why some theoretical concepts permit of such a vast range of deduction, while others are less significant in this respect. We are becoming able to face as never before the logical question as to what we mean when we define facts as being quantitative at all. And as our view of the forms of conceptual structure which are possible for the human mind not only enlarges, but becomes more exact, we are coming nearer to the point where we can profitably study what the conditions are upon which the formation of exact concepts depends.

III.

The researches to which I refer are well known to all students of modern logic. They have come, to a considerable extent, from the mathematical side. They have been suggested, however, not only by mathematical science, but by the logical analysis of the exact physical sciences, and to some extent by the analysis of the concepts which lie at the basis of the study of the humanities, and of the historical sciences. The interest in formal logic which received a new impetus from the researches of Boole, has added itself to these other motives. As examples of inquiry of the type that I here have in mind, one may mention the well known works of Mach, and of Pearson, on the concepts and methods of physical and of statistical science, the recent books of Ostwald and of Poincarè, the various lectures on the concepts and methods of science, which were called out by the St. Louis Congress, the varied and extensive investigations of our principal American logician, Mr. Charles Peirce, the great literature which has now grown up about the theory of assemblages wihch Cantor initiated, the investigations of Dedekind upon the concepts of arithmetic, the lectures and essays of Helmholtz regarding the concepts of the exact natural science, the extensive inquiries into principles of geometry, the modern effort to formulate the concepts and purposes of historical science, the manifold controversies

concerning the office and conceptions of recent psychology, the whole range of researches in modern group theory; and in brief, all the more enlightened types of recent reflection upon the principles of science. Although myself a student of philosophy, I lay here no stress upon the contributions to this research which have in my opinion been due to the progress of modern philosophy viewed as such. There is no reason to consider the philosophers in this field as either a privileged or a dangerous class, or as for that matter easily a separable class. Cantor and Dedekind are philosophers amongst the mathematicians. I suppose it might be fair to call Mr. Bertrand Russell mathematician amongst the philosophers. I am certain that Mr. Charles Peirce is a philosopher. I am certain that Boole, although a mathematician, was guided by profoundly philosophical instincts. My interest at this moment is in laying stress upon the fact that the modern study of this subject is confined to no one branch of students, and on the other hand has so far developed that in this field one is no longer confined to the chance observations of this or of that introspective philosopher concerning what he happens to have noted regarding his personal thinking processes. The science which now deals with the morphology of theories, which seeks for their fundamental concepts, which tries to detect what unity there is amongst these concepts, which endeavors to show wherein lies the advantage which certain concepts possess for the purposes of theoretical construction, this whole science, I say, is now no longer a matter of merely private scrutiny, and of personal opinion. It is full of still unsolved problems; but it has a definite method of work. This method, like that of other sciences, is itself at once empirical and theoretical. Empirically the student of logic treats scientific theories as themselves facts which the history of science presents for his inspection. He analyzes these theories to see what their conceptual structure is. A comparative study of theories shows him the prevalence and the importance of certain types of concepts, such for instance as the concept of quantity itself. The student hereupon undertakes to analyze these various concepts into their elements, to detect what their structure is, to describe them as one would describe organisms or solar systems. He then proceeds to ask in what ways the structure of such theories is determined by the nature of human

thinking. To this end he uses means for the analysis of the thinking process which have become accessible only within the last generation. They are the means furnished by a new and now rapidly progressive science called symbolic logic. Not all the actual students of our topic have as yet made use of this instrument of research. Comparatively few are well acquainted with it. But there can be, to the initiated, no doubt of the fundamental importance of this instrument. By means of this and of other instruments of analysis, the modern student is endeavoring to trace thought to its sources, or in more exact language, to see in just what relations we place objects and ideas before us, whenever we undertake to think about such objects and ideas. The thinking process is by no means as monotonous an affair as the ordinary traditional textbooks of logic have depicted. It is worth while to add that the analysis of concepts in which the student of logic is interested is from this point of view very different indeed from a psychological analysis of thinking or from any analysis that could be carried out either by means of direct introspection or by means of the study of language. Whoever is disposed, as some psychologists are, to imagine that logic is a special branch of psychology, may well be invited to make an excursion into modern logic long enough to consider that analysis of the relations amongst the concepts: *and, or,* the concepts of *implication,* and the concept of *negation,* which the recent methods include. Such psychologists are then invited to endeavor to discover by what psychological analysis of the thinking process they could ever detect these relations.

When the analysis of the thinking process is accomplished, so far as that is yet possible, the student of modern logic is next interested in surveying the range of variation to which our theoretical concepts may be subjected. For it is a notable fact that however wide the range of liberty that we give to our thoughts, however free the range of creative activities over which we let ourselves roam, the results in the way of conceptual structure which appear to be accessible, are remarkably limited as to the number of generically distinct types which appear to be open for our consideration. Each one of these types appears, indeed, to involve, as we have already indicated, an infinitude of various exemplifications. But with all this

wealth, the definite structure, the determinate range of variation of fundamental concepts, the distinctly limited list of categories with which the logician apparently has to deal, together constitute one of the most striking results of the investigation. The thought forms, the kinds of conceptual structures which are possible, are certainly not yet thoroughly known, and their range may prove to be very far greater than we yet suspect. But the notable fact is that they appear to be built up upon a few fundamental types, which remind one by analogy of some such natural types as the vertebrate skeleton, or as the type of the insects. With endless variations in detail, each of these great types is built up in its own way, and preserves its morphological identity through its variations. The thought-types are thus not spread out in endless profusion, but apparently have a well-knit organization of their own, wherein a limited range of fundamental types spring from a common root. For instance, I have already referred to the type of structures which modern group theory defines. This type has, to be sure, an infinity of exemplifications; but all these conform to certain simple and fundamental laws. The one theory of groups consequently includes, in a sense, a very large portion of the theory of those conceptual structures which are prominent in modern mathematics. Yet there are systems whose structure is not that of the mathematical group. Their forms, again, vary in ways which we are only just beginning to understand, but which do not seem to exhibit any merely capricious variety. Unity in variety is, then, peculiarly well exhibited in the world of forms.

## IV.

A few of the problems which such a survey of the morphology of the conceptual world, seems to present, may now be mentioned more in detail. That the forms of possible existence which our thought necessarily recognizes, are indeed limited in number, and depend upon as well as exhibit the necessary constitution of our thought, this philosophers long since came to feel. But the effort to enumerate such fundamental types is greatly hindered by our incapacity directly to analyze through any introspective process what the logical structures of our concepts may be. For a concept, that is a fashion of thinking, expresses a characteristic way of behavior

of the mind, a fashion of reacting to our environment. And no simple introspection can tell what such a way of behavior involves. For just as personal character cannot be discovered by looking within, but must express itself in a long and active life, before it can be fathomed, so with the forms of thought. They are methods of activity. A direct reflection does not discover their constitution and relations. These must be judged through an examination of consequences, and through a development of extensive thinking processes. For instance, if you ask a plain man how he gets the idea of number, he will reply, by counting. And he supposes that he knows by direct introspection what counting is. A psychological analysis made under experimental conditions may in many ways further dissect the mental processes which go on when we count. But how remote any such analysis is from a logical comprehension of the form of thought used in counting will become evident only after one has read such discussions as those of Dedekind in his famous essay on whole numbers, or such as Russell's and Whitehead's recent analyses of the relation between the cardinal and ordinal numbers. The relation between the number concept and the concept of quantity is again wholly inaccessible to direct introspection or to psychological experiment. Only an elaborate process of what one might call logic experimentation brings out the relation between the two concepts. The analysis of Peano, or the recent papers of my colleague, Professor Huntington, are instances of such logical experimentation. The process of experimentation in question consists of undertaking to discover what assumptions, or what various sets of assumptions, are sufficient, or are both necessary and sufficient, in order that one may be able to deduce from them the consequences which are already known to be characteristic of whatever concept one happens to be analyzing. Only by such experimentation can one dissect the thought form with which one is dealing.

It follows from our inability to detect by any direct mental inspection what ones of our concepts are the fundamental ones, it follows, I say, that the older philosophers, including Aristotle, were indeed frequently very profound and as far as they went accurate in some of their logical analyses, but could never be exhaustive, in their account of the first principles of our theoretical thinking.

Some of Aristotle's anaylses of such principles do show in fact a wonderful instinct for the essential, a logical depth of comprehension, which remains permanently marvelous as well as instructive. So, for instance, his brief but penetrating analysis of the concept of the Continuum touches upon a problem which brings him into close touch with the inquiries even of a Dedekind; and this fact about Aristotle's view of continuity has well been pointed out by Mr. Peirce.

But at the best these older analyses labored under one presupposition which was long prominent in philosophical textbooks, which was, however, long since rejected by at least one of the most famous modern philosophers, namely Hegel, and which has now become, as I think, a definitively exposed error, deeply rooted as it still is in the popular mind. This was the presupposition that the first principles of theoretical science, the fundamental concepts upon which all theoretical construction depends, are or can be known to the mind in the form of a list of self-evident principles, or of simply unavoidable and obviously necessary concepts. I say the older analyses of theoretical science mainly depend upon supposing a list of self-evident principles to be discoverable, and a list of self-evident concepts to be attainable. Even Locke, empiricist as he was, regarded the self-evident concepts and principles as indeed psychologically due to our experience, but as coming to our consciousness, after once our experience had been matured, in a shape which made them shine by their own light. But the modern logician has learned to see, that the feeling of self-evidence which frequently attends the enunciation of a principle, is commonly an indication that one has not yet learned to analyze the principle. In other words, self-evidence is a suspicious sign. It warns you that you do not yet understand the topic. If you cut a strip of paper and bring the two ends of its together to make a ring, it appears self-evident that any strip of paper must have two sides, and that in order to get from the inside of the ring to the outside of the ring by a movement which keeps your finger, or a pencil, in contact with the paper, you have either to go through the paper, or to go over the edge of the paper. All this seems self-evident or to many people may seem so, until someone shows you

the now well-known one-sided paper ring, made of an ordinary strip of paper, but so made that the two sides form but a single side. In this case the very strip of paper which has but one side, now has but one edge. And thus a universal principle which might, but for such an example, have seemed self-evident, namely the principle that a ring strip of paper must have two sides and two edges, becomes in the light of this principle, simply false; and one's geometrical ideas are hereby enlarged. So long, then, as it is self-evident to you that any ring strip of paper must have two sides, you simply do not understand the forms in question. Another case now very familiar in discussion, another case, I say, of a principle long regarded as self-evident, is the principle that the whole of a collection, must exceed in multitude any part of that collection which may be formed by leaving some of the members of the collection out. But the modern theory of infinite collections is founded upon supposing this principle to be, as it actually is, false for such collections. Thus there are as many powers of two as there are whole numbers, an assertion which follows directly from the definition of a power of two, and from the definition of whole numbers. Yet the powers of two are themselves whole numbers, and are but a portion of the whole numbers, and may be viewed as an extremely small portion in case one judges its size merely by considering what whole numbers are omitted from this collection. Upon self-evidence, then, no theory of the scope of theoretical science can be built up. I do not hesitate to say that there are no self-evident principles. And as myself, in philosophy, what is called an absolutist, that is a believer in the existence of absolute truth, I utter this assertion not in the interests of skepticism, but in the interests of truth. Single truths do not possess self-evidence, just because there are many truths which form a system, wherein each element is dependent for its nature upon its relations to the others. In general the assertion of the self-evidence of single principles has repeatedly been a foe to the progress of civilization, as it is hostile to a genuinely logical understanding of the nature of truth. The assertion of self-evidence has been used to defend almost any bulwark of tyranny from the questionings of beneficent reformers. Not upon self-evidence, therefore, nor upon a list of fundamental verities, each of which shines merely by its

own separate light, can the logical theory of science be founded. In general what we call first principles are such merely in some certain respect, or from some special point of view. Otherwise viewed, these same principles may appear as derived. And to discuss the various ways in which such derivation may be brought to light, is one of the principle problems of modern logical theory.

## V.

The relative accomplishment of such a task in the case of any particular branch of logical theory involves a sort of study which the recent discussion of the logic of geometry, as well as of the logic of number theory, often exemplifies. Instead of setting forth certain self-evident axioms of geometry, or of arithmetic, the modern logical investigator undertakes to do what Russell and Couturat call defining a certain type of space, or a certain type of numbers. This process of definition, also often called the process of definition by postulates, consists substantially in saying: "I am going to describe to you the properties of a certain class of ideal entities. I do not say that these entities exist in the physical world, just as I do not deny that they exist there. But I am going to treat them simply as the entities which conform to the following definition. The definition I will state in the form of a set of principles given in order as first, second, third, and so on. I state the principles, and I define the entities in question as a set of entities such that they conform to these principles. If the principles involve no mutual contradictions, such entities are possible." Thus Dr. Veblen, in his recent essay on the so-called axioms of geometry, states twelve different principles to which certain abstract entities named points are to conform. He does not assert that these principles are self-evident. Since he is talking about purely abstract entities, which are the creatures of his definition, the principles could not be self-evident. They are true only in the sense that the entities defined are said by definition to conform to them. Dr. Veblen then shows that the laws of our ordinary geometry can be deduced from these principles as laws which hold for the defined entities. These principles, then, are sufficient as a basis for geometrical science.

A similar procedure has now become so common in discussions of this modern type, that it needs no further characterization for those who have examined any such researches. Their interest lies not in the founding of scientific theories upon any set of self-evident principles. The interest lies in showing the connection which exists amongst various concepts and principles, and in bringing to pass a logical analysis of the theory in question and of the concepts which this theory involves. No exclusive significance can be attached to any one such investigation. There are numerous, probably very numerous different sets of principles, upon which geometrical science could be founded. How far our experience of space bears out any of these principles by confirming their truth is a matter for the science of nature. Why our experience of space has these characters is ultimately a matter for philosophy. What set of geometrical principles it is convenient to use for the purposes of a textbook of geometry, is a pedagogical matter. Geometry is not deducible from self-evident axioms, since there are no self-evident axioms. Geometry is a theoretical science, since we are not confined to particular observations for our knowledge of space relations, but are acquainted with laws which enable to describe and predict our spacial experiences in general terms. The first principles of this theoretical science can be variously stated. The logical problem lies in understanding the relations that exist amongst these various statements.

Nevertheless, when a large number of theoretical sciences have been treated in this way, when their various concepts have been analyzed from various points of view, and when as is the case the forms or types of concepts which they contain have been shown to be variations of a comparatively limited number of types, such as the series type, the group type, or to speak of a more special instance, the type of the ordinary real numbers, or of the ordinary complex numbers, one is brought in the presence of a further problem which is indeed at the present time the central and characteristic problem of logical theory. It is the problem as to the unity of these forms. Fundamental ideas, in the sense of self-evident concepts and principles, do not exist in scientific

theory. On the other hand, the various inter-dependent truths and concepts of theoretical science appear to form a relatively closed system, where the special forms are infinitely numerous, but where the main types or species are comparatively few. The question in which all students of science ought to be interested, and in which students of philosophy are explicitly interested, is the question as to what common tendency of human activity it is which differentiates itself into all these forms. What a thinker does when he puts facts together, and forms a theory, depends of course upon the nature of the facts, in so far as he is trying to describe them, but it also depends upon the nature of his thought, in so far as he can only do for the purposes of thinking, what appeals to his rational interest, and what solves a thoughtful problem. A thinker, however faithful to his facts he means to be, has his needs as a thinker, and his forms of thought are his ways of satisfying his needs. He cannot merely report facts. He must interpret them. His theories are his interpretations. His world of science is his world as interpreted. It cannot be understood therefore apart from his needs as a thinker. The structure of his theories is the embodiment of these requirements of his own nature as a thinker. That quantitative science, that the principles of geometry, in whatever form they may be stated, that group theory, and that number systems, apply to his world in the regions of which he has a theoretical understanding, all this is due not merely to any outer world, which can exist wholly apart from the thinker,—not merely to such a world, I say,—but certainly also to the nature of the thinker himself. Our study of theoretical science has to be interpreted, then, as a kind of science of a thinker's ways, as an inquiry into what sort of ideal he has, as a study of the meaning of his thoughtful life, of its internal meaning, and of truth, in so far as truth is related to this internal meaning of the thinker. When we find, as we do, that the forms of thought are not endlessly variable, but are reducible to a certain range of generically different conceptual structures, we are therefore led to this question which now we face. To what are these thought forms due? What is their unity?

## VI.

In an address which I was privileged to make before the St. Louis Congress I pointed out a contribution to this problem which had been suggested and in part carried out by Mr. A. B. Kempe. I have since further pursued the research which Mr. Kempe has initiated, and have published my results in a paper entitled "The Relation of the Principles of Logic to the Foundations of Geometry," printed in the *Transactions of the American Mathematical Society*. This is no place to discuss the issues involved in that paper. I want simply to indicate in a very general way one point regarding the kind of result which seems to me to be already in sight, although the matter is still very incompletely worked out. The different characteristic forms of thought to which I have referred, are distinguished by the various types of relations which these various forms exemplify. Thus the characteristic ordinal relation of descriptive geometry is the relation called "between"; and Dr. Veblen has shown how in terms of this single relation, and of the assumption of the existence of appropriate objects or entities, one could state all the principles that are needed as the foundation for geometry. The characteristic relation of the world of quantity, the relation of "greater and less," is a relation which in combination with the triadic relation that is involved in the ordinary operation of addition, is sufficient to give form to the principles of algebraic analysis. In brief, then, each theoretical science has its own characteristic set of relationships. When so viewed these relations stand by themselves, as if they were separate facts in the natural history of the forms of thought. Relations may be classified, just as truly as birds, or as bacteria may be classified. There are relations dyadic, triadic, n-adic, there are relations symmetrical, unsymmetrical transitive, intransitive. These varieties of form in the world of relation, when thus viewed, seem ultimate and irreducible. Yet I do not think that anybody finds it self-evident, or axiomatic, that only these relations should be possible. I do not think that we have any warrant for saying on the other hand that the sorts of relations which exist are capable of a simply limitless and a capricious variety. The concept of a relation is to my mind, as to the minds of a good many of my colleagues, something that is intelligible only in terms

of the activities of our own thought. We understand relations, because of our own thinking processes we can at once depict, and in a sense reconstruct or create them. The types of our own construction, of our own thoughtful activity, are therefore the relational types. If we are to understand, then, the unity and the system of relational types, we must see how their varieties are related to our own activities as thinkers.

Now, however, relations are known to us not only as existing in teh world of numbers and of geometry, but as present in the purely logical world, the world of classes and propositions, of syllogisms, and of reasonings in general. I have already mentioned what some of the logical relations are. They are relations such as are expressed by the words "*and*," "*or*," "*not*," "*implies*," and so on. These relations are as fundamental and as simple as are our thinking processes themselves. We learn about them not through our senses, but through our activity as thinkers. Now what Kempe's research suggests, and what my own line of research has tended I hope to bring a very small step further on the road towards definition, and confirmation, is the thought that such geometrical relations as "between," such relations as "greater" and "less," and even such relations as are fundamental in group theory, are capable of being interpreted as instances, as consequences, or as partial views, of the fundamental logical relations themselves. Kempe has shown how a logical class can be viewed as "between" two other classes and how the geometrical "between" can be regarded as a special instance of this logical "between" I have shown how the system of Dr. Veblen's principles of geometry could be brought into definite connection with the relations which characterize a system of logical classes. The whole research in question is still in a very elementary stage, but enough has been done, I think, to make it at least probable that whoever comprehends the most fundamental logical relations, such as a child begins to comprehend when it first says "no," that is, whoever comprehends such relations, as "and" and "or" and "not," and the relation of implication, has already in his hand the means for developing the fundamental concepts of all of the exact sciences, since the relations of these exact sciences are more or less

complex variations and recombinations of the fundamental logical relations themselves.

Meanwhile, however, the fundamental logical relations are characteristic not only of our world of thought, but also of our world of action. For will-acts involve acceptance and refusal, affirmation and negation, a consciousness of consequences, a facing of alternatives, a union of various acts in one act; so that the logic of action is in form precisely the same as the logic of abstract thought. In brief, so far as I can see, the trend of the modern study of the principles of theoretical science is at present towards proving that all the forms of conception used in exact science are but expressions of the characteristic types of will activity of which we as voluntary agents are capable. We thus conceive the structure of the world in terms of the structure of our own types of voluntary activity. The forms of our will determine the types of our theoretical concepts. We define facts, so far as we theoretically comprehend them, in terms of the nature of our wills. The view of the logical source, and of the internal structure of our concepts which is thus suggested, is closely akin to what is nowadays called pragmatism. But to my mind any pragmatism rationally thought out becomes philosophically speaking an absolutism. Yet with that philosophical question we have here nothing to do. The result of our modern study of logic is certainly to give us no less respect for facts, than we get from the study of nature. But the facts with which the logican has to deal is the fact that as a man willeth, not only so is he, but such are his theoretical conceptions. The whole trend of his theoretical science consists in his effort to find in the universe, in the end, the expression of his own will. His fancies, his capricious will, his temporary hopes and hypotheses, he learns to resign; and he calls this resignation a submission to external facts. But this submission itself is an action of the will, a rational act, but also his own act. As he proceeds in the work of his thinking, he is, as Kant long ago said, endless interpreting the world in terms of his own thought. But the forms of his thought, these prove to be ultimately the forms of his voluntary activity. Our modern unification of the concepts of theoretical science looks then towards viewing all the fundamental types of relations as identical with the types of the purely logical relations,

such as come to mind when we assert, deny, infer, or otherwise deal with the relations expressed by the words "and," "or," "not," "implies" and a few similar terms. But these forms of relations are themselves the forms in which our will embodies itself. So that our theories of the universe tend to be like the other works of our civilization, the result of a long struggle with nature, by means of which, when we win at all, we attain the end of finding our own will expressed in the order of the controllable facts. Some such consideration the modern study of the principles of theoretical science seems to me to enforce; and from this point of view I regard this study as belonging to what Franklin had in mind when he used the term "useful knowledge."

CAMBRIDGE, MASS.,
April 18, 1906.